to conjoin and form uninterrupted strata; they are strictly argillo-ferruginous lime-stones. No others, however, are equal to those from Sheppy; and so great is the demand, that much of what is made and sold as Roman cement, at half the original price of that article, is very little entitled to the name; although it can still be had as good as ever at the original patentee's, but at a more expensive rate. As originally made, it might be used during a sharp frost, which would not be safely practicable with the generality now made. The greater part of that now used is from Harwich, and when burnt by itself being much darker than the genuine cement, is frequently, by the manufacturers, depreciated in quality by the admixture of Swalecliff and other light-coloured stones, which render judgment of it by colour almost impossible. The Swalecliff stone, while it gives it the property of setting quickly, like Sheppy cement, is said to make it very liable to give way afterwards. On being analized, the Harwich cement was found to contain carbonate of lime 70, aluminous earth 24, and iron 6 per cent. A bushel of the stone broken small, ready for burning, weighed 118lbs., when burnt, 82lbs., and when ground, 88. 10 bushels of stone produce about 11 of cement, exclusive of waste: the best, when mixed up ready for use, is of a dusky-green colour, and being so dark, requires frequent colouring. Lime-stones are found in small quantity in the marshes of Essex, near Steeple, affording a cement which is stated to be little inferior to that of the Sheppy stones.

35. An excellent mortar is composed of one part Harwich cement, one part ground quicklime, and one part sand; these ingredients are well mixed through a sieve; and the mortar is made in a mortar-mill, if conveniently it may, the water being added gradually until the whole is perfectly incorporated, and a due pasty consistency is obtained. It is easy to work, and slower in setting than cement made without lime; but it becomes very hard in a few days, and soon acquires great strength.

(To be continued in our next.)

POPULAR OUTLINE OF THE QUADRATURE OF THE CIRCLE.

BY OLIVER BYRNE,

Late Professor of Mathematics to the College for Civil Engineers; author of "The New and Improved System of Logarithms;" "The Doctrine of Proportion;" "The Practical, Complete, and Correct Gager;" "The Elements of Euclid by Colours;" "A Practical Treatise on Spherical Trigonometry."

THE quadrature of the circle cannot be at variance with the strict character of a publication like THE BUILDER, as its chief designs are to spread information and discuss difficulties, which are interesting to all general readers of subjects of architecture and its kindred arts and sciences.

However, the few remarks here offered are given more with a view to gratify curiosity, than to afford any additional information on a subject so much discussed by the scientific of all ages, and long since proved to the satisfaction of reasonable men to present impossibilities, which, as such, it would be ridiculous with any existing method to attempt to overcome. The exact ratio which the diameter of a circle bears to its circumference has never been determined. This celebrated problem, called *squaring the circle*, has for ages exercised the abilities of the greatest mathematicians. Many persons of eminence have at various times laid claim to the honour of having achieved the task, but their errors have been soon detected: among these were Longomontanus, and our own countryman Hobbs; the latter steadfastly insisted that he had done so, nor would he bear any demonstration to the contrary. The world always contains a great number of such men as Hobbs, who not only deceive the mass of mankind for a while, but very often impose upon themselves.

Though the problem is apparently very simple, yet its solution defies the art of analysis, and is now generally considered an impossibility; and indeed the consideration is justly formed, if the attempt be made by any method known at present.

But although the exact relation between the diameter and circumference cannot be expressed in known possible quantities, an approximation to the truth may be made to any assigned degree of exactness. In this manner the problem was solved by Archimedes, who, about two thousand years ago, discovered that the diameter is to the circumference as 7 is to 22, nearly. The proportion of Vieta is that of 113 to 355, which is more exact than that of 7 to 22, for it is true to six decimal places. It was derived from the pretended quadrature of Van Eick. This proportion is readily retained in the memory; for, if we set down two units, two threes, and two fives, and divide the digits into two thus:—113 : 355 is the ratio.

But the first person who ascertained the ratio to any degree of exactness was Ludolph Van Ceulen, a Dutchman, who published it in his book *de Circulo et Adscriptis*. He found that if the diameter of a circle be 1, the circumference will be 3·14159265358979323846264338327950288 nearly; which is exactly true to 36 places of decimals; and was effected by means of the continual bisection of an arc of a circle, a method so exceedingly tedious and laborious, that its accomplishment would occupy a considerable portion of a man's life. This achievement was at the time so much applauded, that its result was carved in figures as above on Van Ceulen's tombstone.

The industrious Abraham Sharp (amanuensis to Newton) extended the calculation to upwards of seventy places of decimals. Mr. Machin, who was Professor of Geometry in Gresham College, London, calculated the quadrature of the circle true to a hundred places of decimals; it is needless to give the figures, as they are merely a continuation of those obtained by Van Ceulen. Mons. de Lagney, Euler, and others, have continued it still further; but should we attempt to give an account of all the mathematicians who laboured at this celebrated problem, we should very far exceed our proper limits. Among the ancients were Archimedes, Thales, Anaxagoras, Philo, Brayson, Antiphon, Plauto, Appollonis, Ptolemy, Vieta, and many others. With all the late improvements in science to assist them, we find among the moderns the names of Newton, Cavalerius, Dr. Willis, Mr. Huygens, Mr. J. Gregory, Mr. Edmond Hally, &c.; but in this learned age, and in the enlightened United States, we have a Mr. Young, who, according to his own account, has proved all former mathematicians in error; we believe there is one exception, Mr. Hall, of King's College, whom Mr. Young allows to be his equal. Though the equality of curves to straight lines is a subject that many of the first-rate mathematicians among the ancients were very solicitous about, yet nothing very considerable was done until the year 1657, when two Irishmen, Mr. Neil and Lord Brounker, independently of each other, demonstrated the quadrature of some curves. Soon after this, Sir Christopher Wren and Mr. Huygens contested as to the priority of having found the quadrature of a cycloial space.

But the first who brought quadrature under an analytical calculus was Mercator, in 1668, when he published a demonstration of Brounker's quadrature of the Hyperbola. It is curious to see how great men will sometimes descend from their justly-acquired dignity: we find Dr. Wallis taking praise to himself for dressing up Brounker's continued fractions; and even Newton could not be satisfied with his great improvement on Mercator, but claimed the whole discovery.

The same magnitude may be expressed by several different series. If the diameter of a circle be *one*, its circumference will be an infinite series of fractions, each numerator being always four, the denominators in the natural series of uneven numbers; and all these terms to infinity, alternately too great and too little. Newton and Leibnitz gave two other series, apparently very simple, and many more might be added, but they are of a similar nature. Could any of these series be summed, *the quadrature of the circle* would be found; but this has not yet been done, nor is it at all probable it ever will be so. Even from what has been said we learn one great philosophical fact, i. e., that the extent of our knowledge comes not only short of the reality of things, but even of the extent of our own ideas. We have the ideas of a square, of a circle, and of equality, and yet perhaps shall never be able to find a square equal to a circle. But while we assert the uselessness of all attempts hitherto made, let us not be understood to mean that a method will not yet be invented which may set the matter satisfactorily at rest for ever; a new relation of quantities may be discovered, immense improvements will probably be made in the summation of infinite series, or even a new system of notation may yet be invented, that will render this *ratio* determinate and finite: for it is impossible even to conceive the limits to which science, in its vast advancements, may extend. However, if the exact ratio could be obtained, it would be a sort of mathematical triumph rather than a real good, for any one ratio which we have given is sufficiently accurate for all practical purposes. To use the words of the justly-celebrated Charles, afterwards Dr. Hutton, whose opinions we greatly respect: "Although a space be not quadruple by the methods yet known, it does not therefore follow that its quadrature is impossible, or that some method may not hereafter be discovered by which it may be squared."

"All the methods used by geometers before Archimedes were insufficient for the quadrature of any curved space whatever; but were they therefore to infer that no curve could by any means be squared? What has since been done abundantly shews the imprudence and falsehood of the assertion. Archimedes discovered a method by which he squared the parabole; and (save the doctor) by the lately discovered method of fluctions we can find as many quadrable curves as we please. But whilst I am urging the possibility of the quadrature of any space, I am not ignorant of the pretensions of several people to prove the impossibility of that of the circle in particular. There are attempts to demonstrate this impossibility in the Leipsic acts as well as in our Philosophical transactions; but these demonstrations are far from being sufficiently general to afford any conviction of it." It may here be remarked, that Des Cartes, in particular, insisted on the impossibility, on the ground that a right line and a *circle*, being of different natures, there can be no strict proportion between them.

In conclusion we may add, who can prove that even the root of *two*, or any other number whose root in the present scale is an infinite series, may not be a terminable quantity in some scale whose root is a square number, for such it must be? It is true we have not yet found the area of a circle, the diagonal of a square, or the root of 2, &c., infinite terms, yet for each of these we can assign infinite series, whose laws of progression are visible, which are more than the ancients could do, or perhaps expected could be done, if they ever at all thought of such things. And I have no doubt that hereafter will be discovered a method of squaring any figure whatever, which is the chief problem in geometry.

ROYAL PALACES, GARDENS, &c.—Of an account of the royal palaces, gardens, parks, &c., from the year 1838 to 1842, both inclusive, the following are the results:—In the year 1842-43, the sums received were as follows—viz from Hyde, St. James's, and the Green parks, 1,089*l*; from Kensington gardens, 70*l*.; Regent's park, 196*l*.; Greenwich park, 6*l*.; Kew gardens, 1,118*l*.; Old deer park at Kew, 1,170*l*; Richmond park, 726*l*.; Hampton court and Bushey parks, *nil*; Windsor Great park, 5,330*l*.; New Royal kitchen gardens at Frogmore, 73*l*.; the Phœnix park in Dublin, 1,720*l*., and the sum of 2,000*l*., being a portion of the purchase-money received from Prince Albert, for the stock of the Norfolk sand Flemish farms (purchased on behalf of the Crown in 1837, of the executors of King William IV., being part of his late Majesty's private property). Thus, the total amount received was about 14,298*l*. The account "in detail" shews this revenue to arise principally from the sale of decayed timber, loppings of trees, &c., the sale of hay, bark, stock, garden produce, and old materials; the rent of grass-lands, chairs, and free-board rents; subscriptions for keys of the pleasure-grounds, &c. The sum of 1,170*l*. is paid by the King of Hanover for the rent of the old Deer Park and Kew; and the sum of 800*l*. is annually allowed from the Lord Steward's department towards the expenses of the very valuable Botanical Garden at Kew.

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